

Scale-invariance of human EEG signals in sleep

Shi-Min Cai¹, Zhao-Hui Jiang¹, Tao Zhou^{1,2,*}, Pei-Ling Zhou¹, Hui-Jie Yang², and Bing-Hong Wang²

¹Department of Electronic Science and Technology,

University of Science and Technology of China, Hefei Anhui, 230026, PR China

²Department of Modern Physics, University of Science and Technology of China, Hefei Anhui, 230026, PR China

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We investigate the dynamical properties of electroencephalogram (EEG) signals of human in sleep. By using a modified random walk method, We demonstrate that the scale-invariance is embedded in EEG signals after a detrending procedure. Further more, we study the dynamical evolution of probability density function (PDF) of the detrended EEG signals by nonextensive statistical modeling. It displays scale-independent property, which is markedly different from the turbulent-like scale-dependent PDF evolution.

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-Introduction. The analysis of electroencephalogram (EEG) signals attracts extensive attentions from various research fields, since it can not only help us to understand the dynamical mechanism of human brain activities, but also be potentially useful in clinics as a criterion of some neural diseases. Some previous works have been done on human EEG signals in sleep and other physiological states. In Refs. [1, 2, 3] the correlation dimension and Lyapunov exponent are calculated to characterize and discriminate the sleep stage. Lee *et al.* provides the evidence of the long-range power law correlations embedded in EEG signals [4]. The mean scaling exponents are distinguished according to REM (Rapid Eye Movement), Non-REM and awake stage, and gradually increased from stage 1 to stage 2, 3 and 4 in non-REM sleep. Hwa *et al.* found the variable scaling behavior in two regions, and described the topology plots of scaling exponents in this two regions that reveals the spatial structure of the nonlinear electric activity [5]. The random matrix theory is performed to demonstrate the existence of generic and subject-independent features of the ensemble of correlation matrix extracted from human EEG signals [6]. Yuan *et al.* found the similar long-range temporal correlations and power-law distribution of the increment of EEG signals after filtering out the α and β wave [7]. In the present paper, the Tsallis entropy is used to analyze a series of human EEG signals in sleep.

We use the MIT-BIH polysomnography data, which is consist of four-, six- and seven-channel polysomnographic recordings, each with an ECG signal annotated beat-by-beat, and an EEG signal annotated with respect to sleep stages [8]. Records have been sampled at frequency 4 kHz. Sleep stage was annotated at 30s intervals according to the criteria of Rechtschaffen and Kales, denoted by six discrete levels-1, 2, 3, 4 REM and awake (stages 1, 2, 3, 4 belong to non-REM sleep) [9]. In the present analysis, only the samples containing sufficient records (at least no least than five stages) are considered. A representative

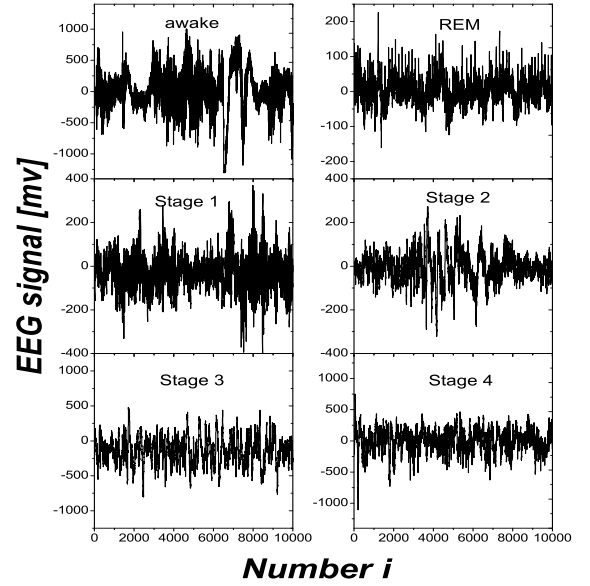


FIG. 1: A set of representative records of EEG signals in different stages. Each entire series includes more than 10^5 data points, while the plotted is only a small fraction.

example is shown in Fig.1.

-Scale-invariance of detrended EEG signals. Consider an EEG series, denoted by $\{x_i\}$ ($i = 1, 2, \dots, N$), whose scaling characteristics are detected through the following procedure:

Step 1: Construct the profile series, $Y_j = \sum_{i=1}^j x_i$, $j = 1, 2, \dots, N$, and consider Y_j as the “walk displacement” of a resultant random walk.

Step 2: Divide the profile series into non-overlapping segments with equal length and fitting each segment with a second order polynomial function. Regard the fitting results as the trends, a stationary series can be obtained by eliminating the trends from the profile series.

Step 3: After the detrending procedure, we define the increment of this modified profile series at a scale s as $\Delta_s Y_j = Y_{j+s}^* - Y_j^*$, where Y_j^* is the deviation from the

*Electronic address: zhutou@ustc.edu

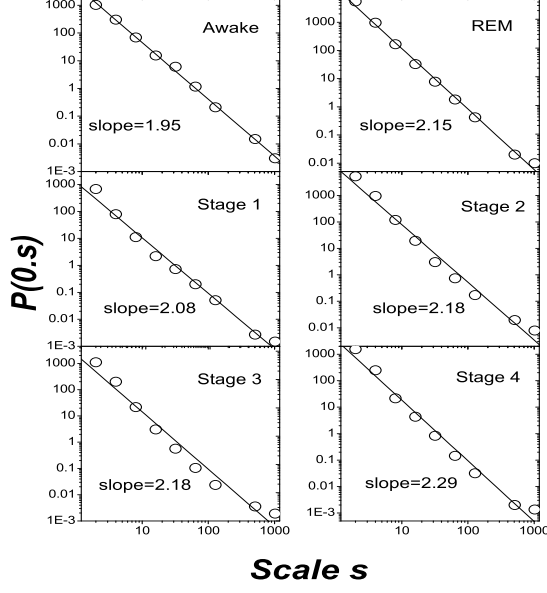


FIG. 2: The probability of $P(0, s)$ as a function of the time sampling scale s . A power-law scaling behavior is observed for about three order of magnitude.

polynomial fit.

step 4: Scale-invariance (self-similarity) in the stationary series implies that the PDF satisfies, $P(x, s) = \frac{1}{\sigma_s} P(\frac{x}{\sigma_s})$, where σ_s denotes the standard deviation at time scale s . Obviously, $P(0, s) = P(0) \frac{1}{\sigma_s}$.

Assigning the values of parameter s from 2^1 to 2^{10} , the normalized PDFs of $\Delta_s Y$ exhibit scale-invariant (self-similar) behaviors as presented in Fig. 2. That is to say, those PDFs can be rescaled into a single master curve, as shown in Fig. 3. The scale-invariance of the detrended EEG signals suggests that the quasi-stationary property is embedded in the distributions of time scales. Therefore, it helps us to search for stable distributions to mimic them.

-Nonextensive statistical modeling of detrended EEG signals. From the results sketched in the preceding section, herein we use the Tsallis entropy to model the PDFs. The Tsallis entropy is introduced by Tsallis through generalizing the standard Boltzmann-Gibbs theory [10], which reads,

$$S_q = k \frac{1 - \int dx [p(x)]^q}{q - 1}, \quad \left(\int dx p(x) = 1; q \in R \right). \quad (1)$$

In the limit $q \rightarrow 1$, S_q degenerates to the usual Boltzmann-Gibbs-Shannon entropy as $S_1 = -\int p(x) \ln[p(x)] dx$. The optimization (e. g. maximize S_q if $q > 0$, and minimize S_q if $q < 0$) of S_q with the normalization condition $\int dx p(x) = 1$, as well as with the

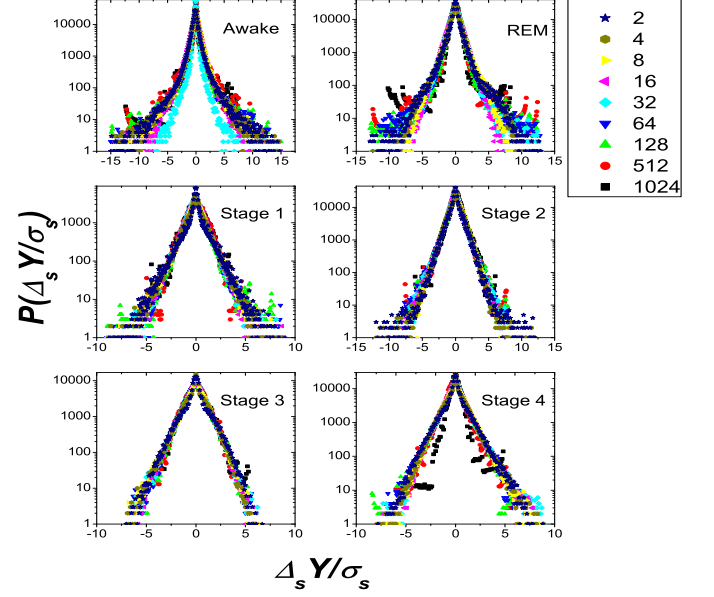


FIG. 3: (color online) The rescaled increment's PDFs for six stages. Obviously, curves with different time scales can well collapse into a single master one, demonstrating the existence of quasi-stationary property.

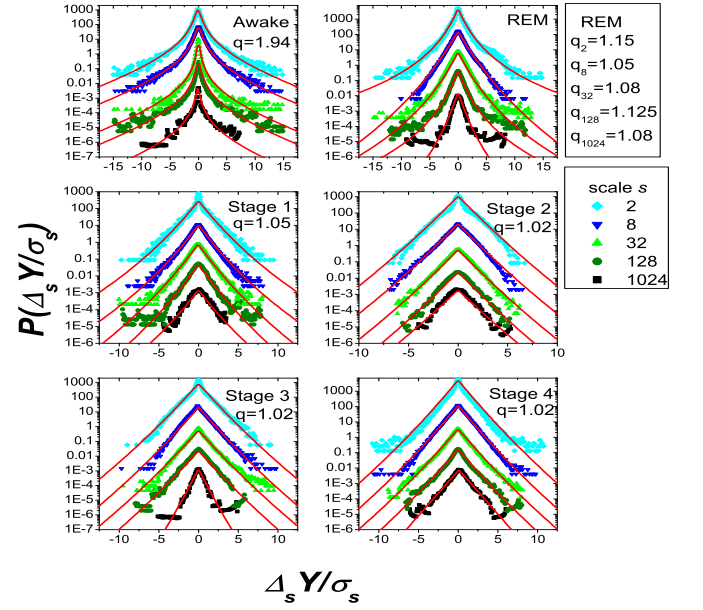


FIG. 4: (color online) The rescaled increment's PDFs of all stages with the approximate fit using nonextensive statistical modeling. We use q -Gaussian function to fit the awake stage, and q -exponential function to fit the other five stages. The awake stage falls into lévy regime with well fitting parameter $q = 1.94$. In the REM stage, the values of q is slight change describing as q_2, \dots, q_{1024} , while in the non-REM stages, parameter q shows a unique value for each specific stages.

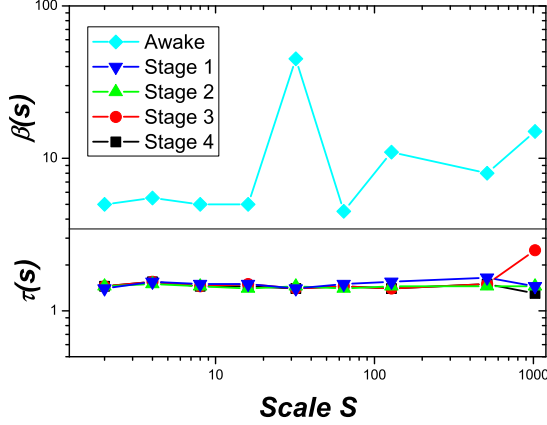


FIG. 5: (color online) $\beta(s)$ and $\tau(s)$ versus s for awake and non-REM stages. The values of $\beta(s)$ don't dissipate as the increasing of s . In particular, $\tau(s)$ of non-REM sleep converge to an invariant pattern.

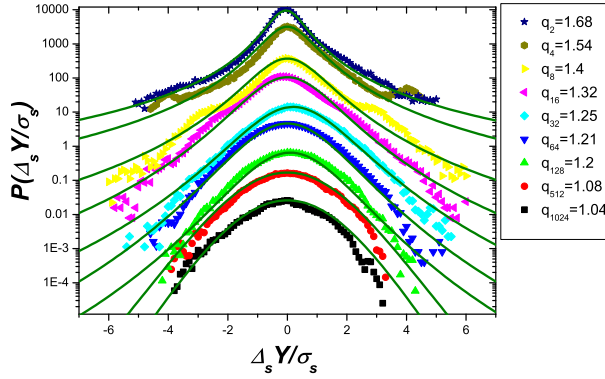


FIG. 6: (color online) The increment's PDF of randomized series of awake stage and fitting curves with different parameter q . The parameter q rapidly approaches to Gaussian regime ($q = 1$) as the time scale increases. For clarity, we shifted the distributions through dividing them by their standard deviation

constrain $\langle x^2 \rangle_q = \sigma^2$, leads to q -Gaussian distribution

$$G_q(x, s) = \frac{1}{Z_q(s)} \{1 - \beta(s) [(1-q)(x - \bar{x}(s))^2]\}_+^{\frac{1}{1-q}}, \quad (q < 3), \quad (2)$$

where $Z_q(s)$ is a normalization constant, $\beta(s)$ is explicitly related to the variance of distribution, and the subscript “+” indicates that $G_q(x, s)$ is non-negative [11]. $G_{q \rightarrow 1}(x, s)$ recovers the usual Gaussian distribution. The q -Gaussian PDF can describe a set of stable distributions from Gaussian to Lévy regimes [12] by adjusting the value of q with appropriate time-dependent parameters $\beta(s)$ and $Z_q(s)$ [13]. The distribution falls into Lévy regimes in the interval $\frac{5}{3} < q < 3$, with $q = 5/3$ a critical

value.

The results in Fig. 4 show us that the PDFs of awake stage falls into the Lévy regime with q being equal to 1.94. It exhibits sharp kurtosis and long-tail distribution, distinguished from those of REM and non-REM stages. It should be noted that we shift the distributions through dividing by their standard deviation and only plot part of them to make the figures clear. The specific values of $\beta(s)$ for all scales are shown in the Fig. 5. It interests us that $\beta(s)$ does not dissipate as the scale increases unlike the case of $\beta(s)$ found in financial market [14]. In other words, It demonstrates that the dynamics evolution of EEG signals is not coincident with the diffusion process described by Fokker-Planck equation.

Another significant equation of nonextensive statistical approach is the q -exponential function, which reads

$$e_q(x, s) = \frac{1}{Z_q(s)} \{1 - \tau(s) [(1-q)|x - \bar{x}(s)|]\}_+^{\frac{1}{1-q}}, \quad (3)$$

where the parameter $\tau(s)$ is the relaxation rate of distribution. Clearly, in the limit $q \rightarrow 1$, $e_1(x, s) = \frac{1}{Z_q(s)} \exp\{-\tau(s)|x - \bar{x}(s)|\}$. Because the statistic distributions of detrended increment of EEG signals in sleep stage exhibit an approximately exponential form, we use the q -exponential model to quantifies them, as shown in Fig. 4. The values of q for the REM and non-REM stage are little larger than 1. It means that the fluctuation of human brain activity in sleep stage will converge to a normal exponential pattern. In particular, the EEG signals exhibit q -invariant pattern for different time scales in all the four stages within non-REM sleep. The relaxation rates of distributions are also approximately invariant, as shown in the Fig. 5. However, in the REM stage, the values of q are slight change because of fitting the tail of distribution in different time scales, and the model can only well fit the center distribution. It suggests that brain electric activity in the REM stage may work in a more complex pattern than awake and non-REM stage for the acute neural activity [15].

The nonextensive statistical approach modeling the detrended increment's PDF of EEG signals with an invariant parameter q demonstrates the scale-independent property of the system. In order to further test the existence of this observed property, we randomize the empirical series of awake stage by shuffling [16, 17] and show a fit for this artificial distributions at different scales in the Fig. 6. Clearly, the parameter q will approach to Gaussian regime ($q = 1$) as the time scale increases. This result strongly supports the scale-independent property of human brain activity in sleep is remarkably different from the turbulent-like scale-dependent evolution [18].

-Conclusion. In this work, several dynamical properties of human EEG signals in sleep are investigated. We firstly use a modified random walk method to construct the profile series including the information of EEG signals. After a detrending procedure, we obtain the stationary series and define the increments of the resultant

random walk at multiple scales. In order to characterize the dynamical process of brain electronic activity, we then study the $P(0, s)$ of the PDF of normalized increments as a function of s . With this choice we investigate the point of each probability distribution that is least affected by the noise introduced by experimental data set. The scale-invariance in both awake and sleep stages are obtained, thus one can rescale the distributions at different scales into a single master curve.

Aim at this property, we use nonextensive statistical approach to model these processes. The dynamical evolution of detrended increment's PDF in awake stage can be well fitted by q -Gaussian distribution with a invariant parameter $q = 1.94$. It demonstrates that the PDFs of awake stage fall into the lévy regime. Contrastively, q -exponential distribution is used to mimic PDFs of sleep stage. In particular, for the non-REM stage, it exhibits scale-independent distributions, while for REM stage, it suggests a complex distributional form with the values of q slightly different.

The statistical properties of distribution of EEG signals strongly indicate that the process of brain electric activity is remarkably different from turbulent-like cascade evolution. In a recent work [18], Lin-Hughson proposed a turbulent-like cascade model to mimic the human heart-rate, whose validity is, now, in the face of challenge on critical scaling-invariance found in real human heart-rate processes [19, 20]. Although the electrocardiograph

(ECG) and electroencephalogram are different, they have some common statistical features. We wish this work is helpful for the in-depth understanding about the underlying dynamical mechanism. And, like the corresponding empirical studies on human ECG singles [19, 20], this work could provide some criterions for theoretical models on human EEG signals.

These similar results are also found in the ECG signals of human. Since the work presents the turbulent-like cascade heart rate model to mimic human heart rate, Kiyono *et al.* found that the human heart rate exhibits critical scaling-invariance, of which the dynamical evolution of increment's PDF is different from turbulent-like PDF evolution.

Further more, some very recent works [22, 23] pointed out that the sleep-wake transitions exhibit a scale-invariant patterns and embed a self-organized criticality (see also Ref. [21] for the concept of self-organized criticality). Thus, the dynamical properties of human EEG signals in sleep suggest that human brain activity in sleep may relate with a self-organized critical system [24]. Our empirical result, in some extent, support those conclusions.

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